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Effect of Predators on the Spread of Hantavirus Infection

(Kesan Pemangsa ke atas Penyebaran Jangkitan Hantavirus)

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ABSTRACT

Hantavirus is a serious disease caused by rodents which can lead to mortality. Many efforts have been carried out by researchers to develop and analyze mathematical models of Hantavirus infection. In this paper, the Peixoto and Abramson (2006) biodiversity model is modified to include the effect of predators and study the prediction of the modified model. When rodent and predator populations are in competition, the predator populations have the effect of reducing the prevalence of infection. Predators may be used for control and reduces the number of competing species to stabilize the populations at a persistent equilibrium.

Keywords: Biodiversity model; competition dynamics; Hantavirus infection; mathematical model; predator-prey model

ABSTRAK

Hantavirus adalah jangkitan serius yang disebabkan oleh tikus yang boleh membawa kepada kematian. Banyak usaha yang telah dilakukan oleh para penyelidik untuk membangun dan mengalisis model matematik jangkitan Hantavirus. Dalam artikel ini, model biodiversiti Peixoto dan Abramson (2006) diubahsuai untuk memasukkan kesan pemangsa dan mengkaji ramalan model. Apabila populasi tikus dan pemangsa dalam persaingan, populasi pemangsa memberi kesan pengurangan kekerapan jangkitan tikus. Pemangsa boleh digunakan untuk mengawal dan mengurangkan bilangan spesies yang bersaing untuk menstabilkan populasi pada keseimbangan yang berterusan.

Kata kunci: jangkitan Hantavirus; model biodiversiti; model matematik; model pemangsa-mangsa; persaingan dinamik

INTRODUCTION

Hantaviruses are viruses carried by certain kinds of rodents and being significant pathogens, can cause hemorrhagic fever with renal syndrome (HFRS) and hantavirus cardiopulmonary syndrome (HCPS).

In 1993, an outbreak of HPS occurred in the South West corner of USA resulting in a high mortaility rate. A basic mathematical model was developed by Abramson and Kenkre (2002) to simulate the spread of the virus and it was found to be able to replicate some features of the infection such as the sporadic disappearance of the infection and the existence of refugias for the rodents when environmental conditions are not favourable for the rodents (lack of water, food and shelter). Related studies on modeling for Hantavirus infection can be found in Abdul Karim et al. (2009), Abramson and Kenkre (2002), Abramson et al. (2003), Giuggioli et al. (2006), Goh et al. (2009) and Yusof et al. (2010).

In a real ecosystem, rodents not only share the living environment with rodents but they have to share with others species. This resulted in inter and intra-species competition for resources (Peixoto & Abramson 2006). Based on the Peixoto and Abramson (2006) research, hantavirus infection is reduced by the influences of biodiversity, in which the alien population tends to reduce or completely eliminate the spread of infection when the alien and rodent populations survive in the ecosystem.

The issue aimed to study was the consequence if the second species or 'alien' is also a predator. According to Campbell et al. (2008), a predator is defined as a species which kills and eats the other animal (prey). This provides check and balance of population in an ecosystem so that the populations within the ecosystem don't exceed the environmental limits. Predators have a range of effects on rodent populations, with some species causing chronic suppression and others causing boom-and-bust cycles or chaos. In interacting systems of predator, prey and pathogen, theory predicts that the loss of predators should tend to increase the absolute and relative number of infected prey (Ostfeld & Holt 2004).

Abramson and Kenkre (2002) have developed a mathematical model, based on differential equations, for the analysis of the spread of Hantavirus infection. The model was able to reproduce the temporal and spatial features that have been observed. The model of Abramson and Kenkre (*AK*) was extended by Peixoto and Abramson (2006) to study the influences of biodiversity the rodents population. Peixoto and Abramson (2006) studied the effect of biodiversity on the prevalence of the infection by using a model in which a non-host population competes with the host i.e. rodents. They showed that an existence of the second species has an important consequence for the prevalence of the infectious agent in the host. When the two rodent species survive in the ecosystem, the competitive pressure of the second species may lead to reduction or complete elimination of the prevalence of the infection. There have also been recent works on predator-prey model by Rafikov et al. (2008). Rafikov et al. (2008) studied the theory of dynamic systems in relation to the mathematical modeling of biological pest control. From the numerical simulations, the results of the Rafikov et al. (2008) model shows that the steady value converges to the equilibrium where both the prey and predator population coexists in the form of a stable equilibrium. The increase in predators will cause the population of the prey to decrease.

Predators are not scavengers or decomposers but they are agents of mortality, feeding on living organisms. Predators can have a great impact on the size of prey populations and can even create population cycles (Smith & Smith 2009). According to Schurr (2003), if the population of the predator species falls, there will be rapid growth in the prey population due to the decrease of predator species. As the prey population increases, starvation rates among the predator species will drop which then causes the predator population to surge. The increase in predators will cause the prey population to decline and the surplus predators will die off due to lack of food and this cycle continues indefinitely.

The first aim of this paper was to conduct numerical experiments on the biodiversity one rodent, one alien model of Peixoto and Abramson (2006) to highlight the salient points of the model. The second aim was to develop and analyse the one rodent, one alien (as predator) model and the third aim was to conduct numerical experiments on the one rodent, one alien (as predator) model.

Mathematical Equations

Consider the basic model of Abramson and Kenkre (2002) which is of the form:

$$\frac{dr_s}{dt} = br - cr_s - \frac{r_s r}{k} - ar_s r_i,$$

$$\frac{dr_i}{dt} = -cr_i - \frac{r_i r}{k} + ar_s r_i,$$

where r_s and r_i are the populations of susceptible and infected rodents, respectively, where $r(t) = r_s(t) + r_i(t)$ is the total population of rodents. For abbreviation, we shall refer to this model as the basic *AK* model.

The value br represents the births of rodents, all of them born vulnerable to the infection at a rate proportional to the total population assuming that all rodents contribute equally to the reproduction process. The value c represents the natural death rate. The infection does not cause deaths among rodents. The value $-\frac{r_s r}{k}$ or $-\frac{r_i r}{k}$ represents a limitation process in the rodent population growth due to competition for resources shared between r_s and r_i. In the basic model, parameter k depends on time and is a 'environmental parameter'. Higher values of the environmental parameter k represents higher availability of water, food, shelter and other resources for the rodents' use to thrive. $ar_s r_i$ represents the number of susceptible rodents that get infected due to an encounter with an infected rodent (e.g. bites from fights) at a rate a (assumed constant). The value a is known as the 'aggression parameter'. Kenkre et al. (2007) states that rodent do not die, nor are impaired, from contraction of the virus. There is no 'vertical transmission' of the disease, i.e. there are no rodents born infected from parents who are infected. Furthermore, humans get the virus from the rodent but, in turn, have no feedback effects on the rodent in the infection process.

According to Abramson and Kenkre (2002), there is a critical value of the environmental parameter $\left(k_c = \frac{b}{a(b-c)}\right)$ that separates two distinctive regimes. If the environmental parameter k is smaller than k_c, r_i tends to zero and the infection dies away. If $k > k_c$, the infection thrives since there is an increase in resources. As the environmental parameter will vary with time, the system will undergo transitions from one state to another.

Peixoto and Abramson (2006) extended the basic *AK* model by including an 'alien' population. The host is identified by the variable r, and the non-host 'alien', identified by z. The competition dynamics Peixoto and Abramson model can be written as:

$$\frac{dr}{dt} = (b-c)r - \frac{r}{k}(r+qz),$$

$$\frac{dz}{dt} = (\beta-\gamma)z - \frac{z}{\kappa}(z+\varepsilon r),$$

where r represents the population of host and z represents the population of alien at any time t. All coefficients are positive constants. The meaning of terms in Peixoto and Abramson model is as follows:

The value $-\frac{r^2}{k}$ represents a limitation process in the rodent population growth. The parameter k depends on time and is a 'environmental parameter' of host in the absence of an alien population ($z = 0$). The value q represents the influence of the alien population and κ represents a environmental parameter of alien population. The parameters β, γ and ε represent the analogous parameters (to b, c, q) for the alien population.

The one rodent one alien model developed by Peixoto and Abramson (2006) is given by:

$$\frac{dr_s}{dt} = br - cr_s - \frac{r_s}{k}(r+qz) - ar_s r_i,$$

$$\frac{dr_i}{dt} = -cr_i - \frac{r_i}{k}(r+qz) + ar_s r_i,$$

$$\frac{dz}{dt} = (\beta-\gamma)z - \frac{z}{\kappa}(z+\varepsilon r),$$

where k represents something different than κ. But in this paper, the same environmental parameters i.e. k and κ are assumed for each species. The values of the parameter k and κ chosen are the same for the population of rodents and alien to ensure both populations can compete and interact equally for resources in their ecological conditions. Thus, the one rodent one alien model becomes:

$$\begin{aligned} \frac{dr_s}{dt} &= br - cr_s - \frac{r_s}{k}\left(r + qz\right) - ar_s r_i, \\ \frac{dr_i}{dt} &= -cr_i - \frac{r_i}{k}\left(r + qz\right) + ar_s r_i, \\ \frac{dz}{dt} &= \left(\beta - \gamma\right)z - \frac{z}{k}\left(z + \varepsilon r\right), \end{aligned} \tag{1}$$

where r_s and r_i are the population of susceptible and infected rodents, respectively, $r(t) = r_s(t) + r_i(t)$ is the total population of rodents, z is the population of alien, a is the transmission rate of the infection, b is birth rate and c is the natural death. The parameter k depends on time and is a 'environmental parameter' in the absence of an alien population and q is the influence of the alien population, z. For the alien population, β, γ, and ε are corresponding parameters.

According to Peixoto and Abramson (2006), there is a critical value of the environmental parameter $\left(k_c = \frac{b}{a(b-c)} + \frac{qz}{b-c}\right)$ that separates three distinctive regimes. If the environmental parameter k is smaller than k_c, the competitor population results greater than the minimum necessary to force the infected subpopulation to extinction. If $k > k_c$, the system has a positive prevalence of infection. The point $k = k_c$ constitutes a critical point of the system, separating two behaviors that qualitatively differ in the stability of the equilibrium of the infected population. When the intensity of the interacting competition is not very high between rodent r and 'alien' population z, $q < 1$ and $\varepsilon < 1$, then coexistence is stable. If the competition is strong, $q > 1$ and $\varepsilon > 1$, bistability occurs: the final state depends on the initial conditions and if $q > 1$ and $\varepsilon < 1$ (or $q < 1$ and $\varepsilon > 1$), only the strong competitor survives.

A further result is the existence of the critical amount of the aliens, a threshold level in the population of competitors, which drives the system completely to a non-infected state. This value is given by:

$$z_c = \frac{ak\left[b-c\right] - b}{aq}.$$

The critical initial value of the alien population that suppresses the spread of the infection is given by:

$$z(0) = \frac{k\left[ar_s(0) - c\right] - r(0)}{q}.$$

The above equation defines a minimum population of competitors that would inhibit the spread of a small outbreak of infection (Peixoto & Abramson 2006).

ONE RODENT ONE ALIEN (PREDATOR)

The work of Peixoto and Abramson (2006) is extended by assuming that the alien is a predator. The model of one rodent one alien (as predator) species is of the form:

$$\begin{aligned} \frac{dr_s}{dt} &= br - cr_s - \frac{r_s}{k}\left(r + qz\right) - ar_s r_i, \\ \frac{dr_i}{dt} &= -cr_i - \frac{r_i}{k}\left(r + qz\right) + ar_s r_i, \\ \frac{dz}{dt} &= -\left(\beta - \gamma\right)z + \varepsilon rz, \end{aligned} \tag{2}$$

where r_s and r_i are the population of susceptible and infected rodents, respectively, $r(t) = r_s(t) + r_i(t)$ is the total population of rodents and z represents the population of predator. For the predator population, β and γ are corresponding parameters and ε is the product of the per-capita rate of predation and the rate of conversing rodent into predator.

The model is developed based on the following one prey one predator Lotka-Volterra model introduced by Rafikov et al. (2008),

$$\begin{aligned} \frac{dr}{dt} &= a_1 r - r\left[b_1 r + c_1 z\right], \\ \frac{dz}{dt} &= -a_2 z + b_2 rz, \end{aligned}$$

where r and z are the populations of the prey and predator (natural enemies) at the time t, respectively. The parameters a_1, b_1, c_1, a_2 and b_2 represent the intrinsic growth rate of prey, coefficient of intraspecific competition, per-capita rate of predation of the predator, death rate of predator and the product of the per-capita rate of predation and the rate of conversing prey into predator, respectively. All parameters are positive constants.

From Peixoto and Abramson model, the host population is maintained and then the population of the alien is modified by replacing with the predator population identified by the variable z.

The result is the one rodent one alien (as predator) model as follows,

$$\begin{aligned} \frac{dr}{dt} &= \left(b - c\right)r - \frac{r}{k}\left(r + qz\right), \\ \frac{dz}{dt} &= -\left(\beta - \gamma\right)z + \varepsilon rz, \end{aligned}$$

where for the host population, b is birth rate, c is the natural death rate, k is the environmental parameter and q is the influence of the predator population. For the predator population, β and γ are the corresponding parameters and ε is the product of the per-capita rate of predation and the rate of conversing rodents into predator.

Suppose an internal classification of the rodent model is used where r_s is the susceptible rodent, r_i is the infected rodent and that r is the total rodent population

$$r(t) = r_s(t) + r_i(t).$$

Then, the result is the one rodent one alien (as predator) model given is presented as in (2).

Model Analysis

The equilibrium values for susceptible, infected and alien (as predator) populations, r_s, r_i and z, respectively, are obtained by letting $\frac{dr_s}{dt}=0$, $\frac{dr_i}{dt}=0$ and $\frac{dz}{dt}=0$ in one alien, one alien (as predator) model of the system (2). The equilibrium of the system (2), namely $E(0, 0, 0)$.

The stability of system (2) around equilibrium is determined by studying the eigenvalues of the characteristic equation at the equilibrium:

The variational matrix of system (2) at the equilibrium is given by:

$$J\left(r_s^*, r_i^*, z^*\right) \begin{vmatrix} b-c-\frac{1}{k}\left(2r_s^*+r_i^*+qz^*\right)-ar_i^* & b-\frac{r_s^*}{k}-ar_s & -\frac{q}{k}r_s^* \\ -\frac{r_i^*}{k}+ar_i^* & -c-\frac{1}{k}\left(r_s^*+2r_i^*+qz^*\right)+ar_s^* & -\frac{q}{k}r_i^* \\ \varepsilon z^* & \varepsilon z^* & -(\beta-\gamma)+\varepsilon\left(r_s^*+r_i^*\right) \end{vmatrix}.$$

When $J(0, 0, 0) = 0$, then the variational matrix of system (2) at E takes the form of

$$J(0,0,0)=\begin{vmatrix} b-c & b & 0 \\ 0 & -c & 0 \\ 0 & 0 & -(\beta-\gamma) \end{vmatrix}.$$

Now the characteristic equation of equilibrium E which is:

$$((b-c)-\lambda)(-c-\lambda)(-(\beta-\gamma)-\lambda)=0.$$

Clearly, $\lambda_2 = -c$ and $\lambda_3 = -(\beta-\gamma)$ are always two negative eigenvalues. Other eigenvalue is given by $\lambda_1 = (b-c)$. Then, λ_1 will be negative if $b < c$. Hence, the equilibrium E is locally asymptotically stable (Gui & Ge 2005). The choice of the parameters, $a, b, c, k, q, \beta, \gamma$, and ε influence the levels of rodents and alien populations at which this equilibrium is achieved.

Numerical Experiments and Discussion of Results

Numerical experiments on the *PA* biodiversity i.e. one rodent, one alien and one rodent, one alien (as predator) models are conducted. Four scenarios will be considered; these scenarios are given by the same values of the parameter q and ε for predator models. The Matlab software ODE45 was used in all of our numerical experiments. The Runge-Kutta method is used to numerically solve the governing system of systems (1), (2). The parameters $a = 0.1$, $b = 1$, $c = 0.6$, $\beta = 1.0$, $\gamma = 0.5$, $q = 0.2$ and $\varepsilon = 0.1$ ($q < 1$ and $\varepsilon < 1$) are used as they were used by Peixoto and Abramson (2006). For all the predator models, the value of the parameter q chosen is the same for the population of susceptible and infected rodents to ensure both populations have the same strength in competition with the predator's population z. Hence, the value $k_c = 25$ represents the critical environmental condition for the basic *AK* model. The value $k = 150$ is used which means the environmental condition is favourable and thus the infection is thriving for basic *AK* model. Meanwhile $z(0) = 160$ is the critical value of the alien population when $t = 0$ in the rodent population. The duration of the simulation results is 20 years.

Figures 1 and 2 show the rodent and alien populations for the case of $k(=150) > k_c$ and $z(=20) < z(0)$ when one rodent, one alien and one rodent, one alien (as predator) models are solved, respectively, using the same initial values (= 50) for r_s, r_i, $z = 20$, $q = 0.2$ and $\varepsilon = 0.1$ ($q < 1$ and $\varepsilon < 1$).

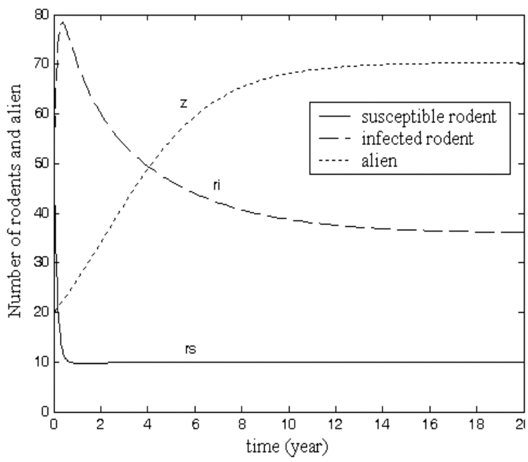

FIGURE 1. Values of r_s, r_i and z for one rodent, one alien model with initial values $r_s = 50$, $r_i = 50$, $z = 20$, $z(0) = 160$, $k = 150$, $q = 0.2$ and $\varepsilon = 0.1$ ($q < 1$ and $\varepsilon < 1$)

For one rodent one alien model, Figure 1 shows that the abundance of resources such as water and food at the initial stage will cause the infected population to increase sharply initially and reaches a certain maximum before plunging down and stabilizing at a steady value of 36 (after 17 years). Since the infection was thriving initially, this reduces the population of susceptible rodents drastically before rising slightly and approaching a stable value when the infected numbers start to stabilize. After 3 years, the susceptible population r_s will eventually stabilize at a steady value of 10. The increase in population of alien z

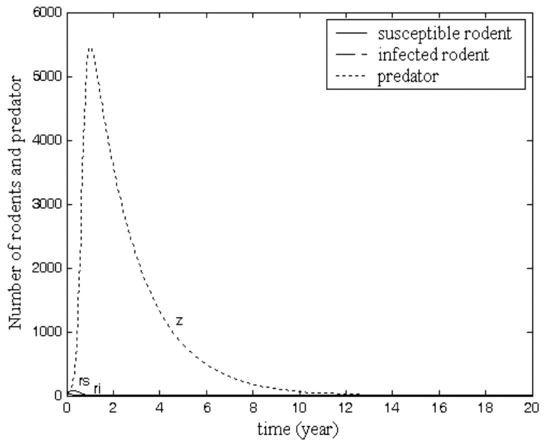

FIGURE 2. Values of r_s, r_i and z for one rodent, one alien (as predator) model with initial values $r_s = 50$, $r_i = 50$, $z = 20$, $z(0) = 160$, $k = 150$, $q = 0.2$ and $\varepsilon = 0.1$ ($q < 1$ and $\varepsilon < 1$)

will initiate the population of susceptible r_s and infected rodents r_i to decrease. This could be due to the population of alien z are better adapted to obtain the resources and can out-compete rodent population and maintain their breeding. After 18 years, the population of alien z will eventually stabilize at a steady value of 70.3. The steady value for alien population z is always higher than the value of rodent population. With $k > k_c$, $q < 1$ and $\varepsilon < 1$, the competition is not very high between rodent population r and alien population z. Thus, the infection will remain. From the simulations, the positive equilibrium (r_s*, r_i*, z*) = (10, 36, 70.3) of system (1) is globally asymptotically stable where the populations of rodents and alien can coexist in a positive equilibrium (r_s*, r_i*, z*). The system can be stable around (r_s*, r_i*, z*) when environmental condition is favorable ($k > k_c$), $z < z(0)$, $q < 1$ and $\varepsilon < 1$.

For the one rodent, one alien (as predator) model, Figure 2 shows the abundance of resources such as food (rodents) at the initial stage will cause the alien (as predator) population z to increase sharply initially and reaches a certain maximum before plunging down and eventually goes to extinction (after 19.6 years). This could be due to the population of alien (as predator) z consume both infected r_i and susceptible rodents r_s. For a long time, there is no food for alien (as predator) z anymore, so the alien (as predator) population z goes extinct. Since the population of alien (as predator) z was thriving initially, this reduces the population of susceptible and infected rodents and the surplus alien (as predator) population z will die off when the susceptible and infected number starts to extinct. After 6.4 years, the susceptible rodents r_s will reduce to zero while the infected rodent r_i become extinct (after 6.6 years). Thus, the infection will die away. The positive equilibrium (0, 0, 0) of system (2) is globally asymptotically stable. Both the rodents and alien (as predator) populations cannot survive and the system converges to the trivial equilibrium (0,0,0). The numerical simulation is given in Figure 2. This clearly indicates that the Hantavirus infection will dies off with the presence of the alien (as predator) populations.

For the one rodent, one alien model, the infected population increases within the first year due to the more abundant resources that the rodents can use to thrive. At the first year, it reaches a certain maximum after which it starts to decrease probably due to the resources being almost used up during the peak population. Meanwhile the susceptible rodent population behaves in the opposite way which is quite expected since more resources mean more rodents are being infected and thus lessen the number of susceptible. For this model, the alien population amplifies caused by the further abundant resources that the alien can employ to flourish. After 18 years, the alien population will then reach steady stable values over some period of time with the number of alien population always exceeding the number of susceptible and infected rodents. In the one rodent, one alien (as predator) model, an increase in food (rodents) availability has only the effect of increasing the alien (as predator) population. After 19.6 years, the alien (as predator) becomes extinct when there is insufficient food (rodents). When $q < 1$ and $\varepsilon < 1$, the competition to survive are very strong between rodents and alien (as predator) populations. What is important here is that the population of alien (as predator) tends to eliminate completely the spread of the Hantavirus infection.

In all of the above models, the steady value of infected rodent r_i is always smaller in the one rodent one alien (as predator) model compared with the one rodent, one alien model. It has the potential to reduce and control the outbreak of a disease. Thus, the introduction of an appropriate number of aliens (as predators) in an infected area may eliminate completely the spread of Hantavirus infection.

Figures 3 and 4 show the rodent and alien populations for the case of k(=150) > k_c and z(=700) >z(0) when one rodent, one alien and one rodent, one alien (as predator) models are solved, respectively, using the same initial values (=50) for r_s, r_i, $z = 700$, $q = 0.2$ and $\varepsilon = 0.1$ ($q < 1$ and $\varepsilon < 1$).

The increase of the initial amount of alien population, displays similar graphical pattern of numerical results. This is due to case of the alien population which is initially small for the one rodent, one alien model which have the same value of r_s but with varying value of r_i and z (by comparing Figure 3 and Figure 1). An increase in water and food availability does not affect susceptible rodent while the steady values for infected rodent and alien populations would change. The infected population r_i will stabilize to steady value of 35.9 after 14 years. Meanwhile the susceptible and alien population will stabilize to steady values of 10 (after 3.2 years) and 70.4 (after 17 years) respectively. The steady value for alien population z is always higher than the values of rodent's population. This could be due to the population of alien z is better adapted to obtain the resources can out-compete rodents population and maintain their breeding. The positive

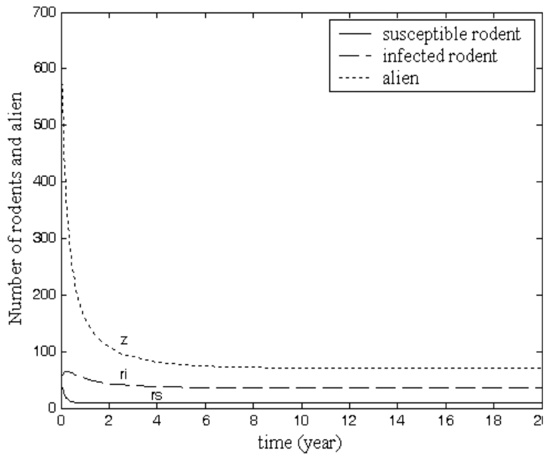

FIGURE 3. Values of r_s, r_i and z for one rodent, one alien model with initial values $r_s = 50$, $r_i = 50$, $z = 700$, $z(0) = 160$, $k = 150$, $q = 0.2$ and $\varepsilon = 0.1$ ($q < 1$ and $\varepsilon < 1$)

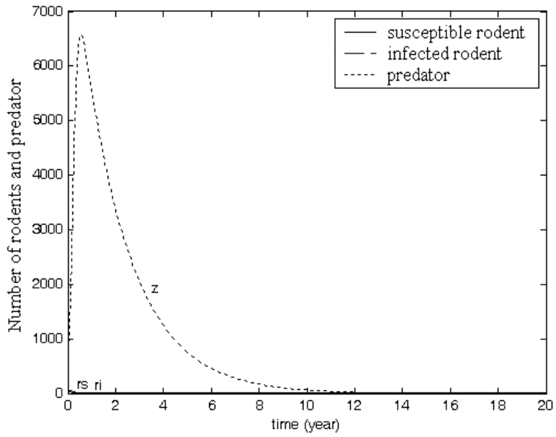

FIGURE 4. Values of r_s, r_i and z for one rodent, one alien (as predator) model with initial values $r_s = 50$, $r_i = 50$, $z = 700$, $z(0) = 160$, $k = 150$, $q = 0.2$ and $\varepsilon = 0.1$ ($q < 1$ and $\varepsilon < 1$)

equilibrium (r_s*, r_i*, z*) = (10, 35.9, 70.4) of system (2) is globally asymptotically stable, all populations can coexist in a positive equilibrium (r_s*, r_i*, z*) when $k > k_c$, $z > z(0)$, $q < 1$ and $\varepsilon < 1$. The numerical simulation is given in Figure 3.

With the increase of the initial amount of alien (as predator) population, the numerical results display similar graphical pattern as the case of the alien (as predator) population is initial small for the one rodent, one alien (as predator) model which have the same values of r_s, r_i and z but with varying value of time t (by comparing Figure 4 and Figure 2). The population of alien (as predator) z is initially large. Moreover the abundance of resources such as food (rodents) at the initial stage will cause the population of alien (as predator) z to increase sharply initially and reaches a certain maximum before plunging down and after a long time, the alien (as predator) z will die off due to the lack of food (rodent). The population of alien (as predator) z will tend to extinct after 19.2 years. However, the increase in z will cause the rodents population to decrease and eventually the rodents population goes to extinction. Both population of susceptible r_s and infected r_i go extinct after 5 years (Figure 4). This situation clearly represents the dynamics of a disease propagate among the alien (as predator). From simulations, the positive equilibrium (0, 0, 0) of system (2) is globally asymptotically stable. The numerical simulation is given in Figure 4. This clearly indicates that the hantavirus infection will dies off with presence the alien (as predator) populations.

All the cases of the one rodent, one alien (as predator) model display all populations are extinct. For all of the above cases, the steady state of values of alien population z is always higher than the corresponding value in the one rodent, one alien model. Moreover, the steady state of values of infected rodents r_i is always slightly smaller than in the one rodent, one alien (as predator) model compared with the corresponding value in one rodent, one alien model. For the favourable condition ($k > k_c$), $z < z(0)$, $z > z(0)$, $q < 1$ and $\varepsilon < 1$, the results of the one rodent, one alien (as predator) model shows the complete elimination of spread of infection hantavirus. With the increase of the initial amount of alien or alien (as predator) population, the steady state of values of infected rodent r_i decrease for the both types of model. It has the potential to reduce and control the outbreak of a disease. However, the alien population is still alive owing to alternative source of food but the alien (as predator) population z becomes extinct occurred by insufficient food (rodents). For the one rodent one alien (as predator) model, the population of infected rodent r_i becomes extinct within the short period of time in the case $q < 1$ and $\varepsilon < 1$. This is due to the alien (as predator) killing and eating all the rodents' population. What is important to note here is that the population of alien (as predator) tends to eliminate the completely spread of Hantavirus infection.

CONCLUSION

In this paper, following the work of Peixotu and Abramson (2006), the effect of predator on the spread of hantavirus infection has been studied. Two different types of models have been analysed; namely, one rodent, one alien and one rodent, one alien (as predator) models. The number of both susceptible and infected rodents was reduced substantially using predation strategy. Interspecific competition can cause population of infected rodent r_i to slowly extinct. The effect of competition between the population of susceptible and infected rodents have been studied together with the populations of alien over a period of time when higher resources are available. For situations where abundant resources are available, the population of alien may reduce the intensity of the infection. These numerical results showed that whether the infection rodent r_i is persistent depends on the initial amount of alien (or predator) population. The increased of the initial amount of alien population, the one rodent, one alien (as predator) model can reduce the hantavirus infection in the infected rodent within a short period of time compared with the one rodent, one alien model. These results suggested the possibility of control of the spread of the epidemic by introducing alien (as predator) in the areas of rodent populations in a suitable way although eliminate infection on the spread of Hantavirus outbreak.

The equilibrium points, existence, stability and numerical experiment of one rodent, one alien and one rodent one alien (as predator) models have been investigated. The stability of positive equilibrium solutions is investigated by linearizing the system of (1), (2) at the positive equilibrium solutions and analyzing the associated eigenvalue problem. The results from the condition for the existence and local stability of positive equilibrium E prove our main result on the stability of the positive equilibrium E of systems (1), (2).

For the one rodent one alien (as predator) model, the system converges to the equilibrium where both susceptible and infected rodents cannot survive in the form of stable equilibrium when $k > k_c$, $z < z(0)$, $z > z(0)$ and ($q < 1$ and $\varepsilon < 1$). The equilibrium points for the one rodent one alien (as predator) model are always smaller than the equilibrium points of the one rodent, one alien model. Thus, the simulation results showed that the equilibrium points for one rodent, one alien model are globally asymptotically stable. For one rodent one alien (as predator) model, all the populations become extinct if $b > c$ when $k > k_c$, $z < z(0)$ and ($q < 1$ and $\varepsilon < 1$). From the numerical simulations, systems (1) and (2) converge to the equilibrium point.

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